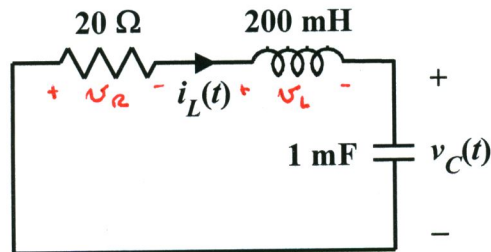# Problem #03

Find $v_C(t)$ for $t \geq 0$ if $v_C(0) = 20$ V and $i_L(0) = 500$ mA.

$$i_L = (1mF)\,\dot{v}_c \quad \Rightarrow \quad \dot{v}_c = 1000\,i_L \quad \Rightarrow \quad \dot{v}_c(0) = 1000\,i_L(0) = 500$$

$$v_R = (20\Omega)\,i_L = 0.02\,\dot{v}_c$$

$$v_L = (200mH)\frac{di_L}{dt} = 0.0002\,\ddot{v}_c$$

$$v_L + v_R + v_c = 0 \quad \Rightarrow \quad 0.0002\,\ddot{v}_c + 0.02\,\dot{v}_c + v_c = 0$$

$$\therefore \quad \ddot{v}_c + 100\,\dot{v}_c + 5000\,v_c = 0$$

$$r^2 + 100r + 5000 = 0$$

$$\Rightarrow \quad (r+50)^2 + 50^2 = 0$$

$$\therefore \quad r = -50 \pm j50$$

$$v_c(t) = e^{-50t}(K_1\cos 50t + K_2\sin 50t)$$

$$\dot{v}_c(t) = -50e^{-50t}(K_1\cos 50t + K_2\sin 50t)$$
$$+ e^{-50t}(-50K_1\sin 50t + 50K_2\cos 50t)$$

$$v_c(0) = K_1 \qquad\qquad = 20 \left.\begin{array}{l} K_1 = 20 \\ \\ K_2 = \dfrac{500 + 50K_2}{50} = 30 \end{array}\right.$$
$$\dot{v}_c(0) = -50K_1 + 50K_2 = 500$$

$$\therefore \quad v_c(t) = e^{-50t}(20\cos 50t + 30\sin 50t) \text{ V}, \quad t \geq 0$$